

Continuum of Fraction Concepts

Van de Walle Developmental Stage, WA State Standards, ORIGO, Investigations, First Steps & selected resources

Math Standard	Lesson Title	Activity overview	Source
Development Stage: Developing the Concept of Fractional Parts/ <i>Sharing and the Concept of Fractional Parts</i> Constructing Fractional Parts <ul style="list-style-type: none"> ▪ <i>Fractional Parts and Fraction words</i> ▪ <i>Sharing Tasks (Van De Walle)</i> First Steps: During Quantifying Phase - During the Quantifying Phase students develop the idea that constructing fair shares requires splitting the whole into equal parts without changing the total quantity and so begin to see the part-whole relations that link sharing to fractions. (<i>First Steps: Diagnostic Map – Number</i>)			
2.4.D Model and describe division situations in which sets are separated into equal parts			<i>Mathematics Standards</i>
2.4.E Interpret a fraction as a number of equal parts of a whole or a set <i>Fractional parts of the whole</i> <i>Equal sized portions or fair shares</i>			<i>Mathematics Standards</i>
	2.4.E 12.1 (15.1) Correct Shares	<i>Whole, one whole, One</i> Fraction words <i>halves, thirds, fourths, fifths</i> . The most important part is discussion of non-examples	<i>Van De Walle</i> <i>Pg.212 (246)</i>
	2.4.E 12.2 (15.2) Finding Fair Shares	Give students models and have them find fifths or eighths or other fractional parts using the models. (The models should never have fractions written on them.) The activity is especially interesting when different wholes can be designated.	<i>Van De Walle</i> <i>Pg. 212 (246)</i>
	2.4.D, 2.4.E, 3.3.A, 3.3.D KU 1 Understanding Fractional Number When we split something into two equal-sized parts, we say we have halved it and that each part is half the original thing.	Students should be encouraged to use a variety of strategies, such as symmetry, dealing out, or measuring to partition quantities into two equal shares.	<i>First Steps in Mathematics: Number, Volume 1, pg 92-103.</i>

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	<p><i>2.4.E, 3.3.A, 3.3.B, 3.3.D</i> KU 2 Understanding Fractional Number We can partition objects and collections into two or more equal-sized parts and the partitioning can be done in different ways.</p>	<ul style="list-style-type: none"> ▪ Equal parts need not look alike, but they must have the same size or amount of the relevant quantity. ▪ When splitting a whole into equal parts, the whole should be completely used up. ▪ Regardless of how we partition, the whole remains the same amount. ▪ The more shares something is split into, the smaller each share is. 	<p><i>First Steps in Mathematics: Number, Volume 1, pg 104-115</i></p>
	<p><i>2.4.E</i> KU 3 Understanding Fractional Number Use fraction-words and symbols to describe parts of a whole. The whole can be an object, a collection or a quantity.</p>	<p>Sample learning activities for beginning, middle, and later stages.</p>	<p><i>First Steps in Mathematics: Number, Volume 1, pg 116-125.</i></p>
	<p><i>2.4.D & 2.4.E</i> Fair Shares (KU 1)</p>	<p>Students work in pairs to equally share a given quantity (using a variety of materials).</p>	<p><i>First Steps in Mathematics: Number, Volume 1, pg 94.</i></p>
	<p><i>2.4.D & 2.4.E</i> Everyday Halves (KU 1)</p>	<p>Students explain the meaning of half in everyday situations.</p>	<p><i>First Steps in Mathematics: Number, Volume 1, pg 94.</i></p>
	<p><i>2.4.D & 2.4E</i> .Halving (KU 1)</p>	<p>Students focus on the act of halving in activities.</p>	<p><i>First Steps in Mathematics: Number, Volume 1, pg 94.</i></p>
	<p><i>2.4.D & 2.4.E</i> Sharing Strategies (KU 1)</p>	<p>Students practice halving objects and collections for a purpose.</p>	<p><i>First Steps in Mathematics: Number, Volume 1, pg 94.</i></p>
	<p><i>2.4.E</i> Chocolate Bars (KU 1)</p>	<p>Students fold or cut several identical rectangular pieces of paper to represent dividing chocolate bars into halves in different ways.</p>	<p><i>First Steps in Mathematics: Number, Volume 1, pg 95.</i></p>
	<p><i>2.4.E & 3.3.A</i> Half a Dozen (KU 1)</p>	<p>Students use an egg carton and eggs to show how six white eggs and six brown eggs could be half a dozen.</p>	<p><i>First Steps in Mathematics: Number, Volume 1, pg 95.</i></p>
	<p><i>2.4.E</i> Half Measures (KU 1)</p>	<p>Students use a half-cup measure to make recipes given in whole-cup measures.</p>	<p><i>First Steps in Mathematics: Number, Volume 1, pg 95.</i></p>

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	2.4.D & 2.4.E Function Box (KU 1)	Students practice halving a collection of objects.	<i>First Steps in Mathematics: Number, Volume 1, pg 95.</i>
	2.4.E Approximate/ Exact (KU 1)	Students identify and list situations in which the word “half” is used. They distinguish between the casual, everyday use of “half” as an approximation, and when the use of “half” is intended to convey an exact quantity.	<i>First Steps in Mathematics: Number, Volume 1, pg 95.</i>
	2.4.E Fair Shares (KU 2)	Brainstorm sharing situations where equality of quantity is important.	<i>First Steps in Mathematics: Number, Volume 1, pg 106.</i>
	2.4.E Segments (KU 2)	Peel citrus fruits. Decide if the segments are equal	<i>First Steps in Mathematics: Number, Volume 1, pg 106.</i>
	2.4.E Making a Sandwich (KU 2)	Students cut a “sandwich” (square piece of paper into fourths and prove that the pieces are the same size.	<i>First Steps in Mathematics: Number, Volume 1, pg 106.</i>
	2.4.E & 3.3.A Sharing Collections (KU 2)	Students share collections that can be easily subdivided.	<i>First Steps in Mathematics: Number, Volume 1, pg 106.</i>
	2.4.E & 3.3.A Equal Shares (KU 2)	Read familiar stories, such as the <i>The Doorbell Rang</i> where sharing is part of the story. Students figure out how many equal shares are needed.	<i>First Steps in Mathematics: Number, Volume 1, pg 107.</i>
	2.4.E & 3.3.A Equal Pieces (KU 2)	Students explore different ways to break a piece of licorice into three equal pieces, then five equal pieces.	<i>First Steps in Mathematics: Number, Volume 1, pg 107.</i>
	2.4.E & 3.3.A Sharing Equally (KU 2)	A group of four students share modeling clay equally.	<i>First Steps in Mathematics: Number, Volume 1, pg 107.</i>
	2.4.E & 3.3.A Thirds (KU 2)	A group of three students share a jug of water equally.	<i>First Steps in Mathematics: Number, Volume 1, pg 107.</i>
	2.4.E & 3.3.A Party Baskets (KU 2)	Students investigate ways of sharing different numbers of sweets among different numbers of party baskets.	<i>First Steps in Mathematics: Number, Volume 1, pg 107.</i>

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	2.4.E & 3.3.A Grouping (KU 2)	Give students collections of discrete item that cannot be cut up.	<i>First Steps in Mathematics: Number, Volume 1, pg 107.</i>
	2.4.D & 2.4.E Fractional Language (KU 3)	Introduce and use fractional language incidentally during construction and play to describe how much is left or has been used.	<i>First Steps in Mathematics: Number, Volume 1, pg 118.</i>
	2.4.D Equal Shares (KU 3)	Students distribute materials within their groups in equal shares. Use of fractional language is a focus.	<i>First Steps in Mathematics: Number, Volume 1, pg 118.</i>
	2.4.D One Third (KU 3)	Students make a “Third” book or classroom display featuring a wide range of materials.	<i>First Steps in Mathematics: Number, Volume 1, pg 118.</i>
	2.4.E Giant’s Jump (KU 3)	Students judge how far they can jump in relation to a ‘giant’s’ jump.	<i>First Steps in Mathematics: Number, Volume 1, pg 119.</i>
	2.4.E Half Full (KU 3)	Use fractional language incidentally when describing how much is left or has been used.	<i>First Steps in Mathematics: Number, Volume 1, pg 119.</i>
	2.4.E Half/Quarter (KU 3)	Use the fractional language of ‘half’ and ‘quarter’ regularly in conjunction with whole numbers.	<i>First Steps in Mathematics: Number, Volume 1, pg 119.</i>
	2.4.D & 2.4.E What Is the Whole? (KU 3)	Students are presented with problems where they must identify the whole when given a fractional part.	<i>First Steps in Mathematics: Number, Volume 1, pg 120.</i>
	2.4.D & 2.4.E Finding Fractions (KU 3)	Students are given a variety of ‘wholes’ and asked to find a given fraction of each whole.	<i>First Steps in Mathematics: Number, Volume 1, pg 120.</i>
	2.4.D & 2.4.E Equal Groups (KU 3)	Students partition a box of paperclips into equal groups. For each group size students name the unit fraction represented by one group.	<i>First Steps in Mathematics: Number, Volume 1, pg 120.</i>
	2.4.D & 2.4.E Fractional Language (KU 3)	Encourage students to use fractional language in response to everyday questions.	<i>First Steps in Mathematics: Number, Volume 1, pg 120.</i>

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	2.4.D & 2.4.E Sharing Sweets (KU 3)	Students are placed into different sized groups, then asked to share equally various quantities of sweets. Students share their results using diagrams and fractional language.	<i>First Steps in Mathematics: Number, Volume 1, pg 123.</i>
	2.4.E Understanding Fractions (KU 3)	Students are invited to explain to the class what they think the term 'fraction' means and to illustrate what they understand about a particular fraction.	<i>First Steps in Mathematics: Number, Volume 1, pg 124.</i>
	2.4.E Counting by Fractions (KU 5)	Have students carry out counting activities in fractional amounts	<i>First Steps in Mathematics: Number, Volume 1, pg 136.</i>
	2.4.E Sharing (KU 6)	Students solve practical sharing problems in which a smaller number of objects is shared among a larger number of people.	<i>First Steps in Mathematics: Number, Volume 1, pg 146.</i>
	2.4.E & 4.2.E Fraction Number Line (KU 5)	Draw a number line on the ground or on a large sheet of paper with units and half units marked. Have students jump in units, half units and/or quarter units, counting as they go.	<i>First Steps in Mathematics: Number, Volume 1, pg 138.</i>
	2.4.E <i>Investigation 1: Shapes Halves, and Symmetry Seeing Shapes Within Shapes 1.2 and 1.3 (min. 1 hr.)</i>	Fitting shapes together to cover a region Putting parts together to find a whole	<i>Investigations Shapes, Halves, and Symmetry, 2nd pg 14-21.</i>
	2.4.E <i>Investigation 3: Fractions of Geometric Shapes Sessions: 3.1 and 3.2 Halves of Rectangles and Solids (min. 2 hours)</i>	Constructing 2-D arrays that divide into halves Constructing arrays to represent numbers and identifying halves of the arrays Representing rectangles using two colors to show halves and not halves Investigating halves with 3-D solids	<i>Investigations Shapes, Halves, and Symmetry, 2nd pg 74-81.</i>
	2.4.E <i>Investigation 3: Fractions of Geometric Shapes Sessions 3.3, 3.4, 3.5 Cutting Congruent Halves (min. 3 hrs)</i>	Folding and cutting shapes into congruent halves.	<i>Investigations Shapes, Halves, and Symmetry, 2nd, pg 82-85.</i>

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	2.4.E <i>Investigation3:Fraction of Geometric Shapes</i> <i>Session3.6 Fraction Flags (min. 1 hr.)</i>	Constructing halves	<i>Investigations Shapes, Halves, and Symmetry, 2nd pg 86.</i>
	2.4.E <i>Fractions of Geometric Shapes 3.7 and 3.8</i> <i>Fourths and Thirds of Rectangles (min. 2 hrs.)</i>	Constructing 2-D arrays that are divided into thirds and fourths Describing fractional parts of an array as fractions of a rectangular region.	<i>Investigations Shapes, Halves, and Symmetry, 2nd pg 89-93.</i>
	2.4.E <i>Investigation 1: Sharing Brownies Session 1-2 Making Fair Shares</i> <i>(min. 2 hours)</i>	Fractional parts of a whole. Fractional parts must be equal. Conventional fraction notation. Becoming familiar with grouping unit fractions, $1/6 + 1/6 + 1/6 = 3/6$	<i>Investigations: Fair Shares, 3rd grade, pg 4-11</i>
	2.4.E <i>Seeing Fractions Among Ourselves</i>	To recognize and name fractions as representative of a counted part of a group or set of objects. To understand the meaning of numerator and denominator in the symbolic form of a fraction.	<i>Fabulous Fractions, AIMS pg. 1-3</i>
	2.4.E <i>Halving Squares</i>	Students explore different ways to divide squares into halves. Students must justify how they are sure that the section they shade is one-half.	<i>Lessons for Introducing Fractions 4-5, pg 75-81.</i>
	2.4.E <i>Sharing Cookies</i>	Students share “cookies” (circle model) among 3, 4, and 6 people. Opportunity to see relationships among halves, thirds, fourths, and sixths.	<i>Lessons for Introducing Fractions 4-5, pg 82-96.</i>
	2.4.E <i>How Much is Blue?</i>	Using 13 pattern block pieces, students solve, “What fractional part of this design is blue?” The challenge for students is that they must think about both the fractional parts of the total design and the areas of the individual pattern pieces.	<i>Lessons for Introducing Fractions 4-5, pg 97-104.</i>

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	<p><i>2.4.E</i> Fraction Scavenger Hunt</p>	<p>To be able to illustrate in multiple ways how a given fraction can be represented as a part of a group or set of objects.</p>	<p><i>Fabulous Fractions, AIMS pg. 4</i></p>
<p>Constructing Fractional Parts</p> <ul style="list-style-type: none"> ▪ Counting Fractional Parts ▪ <i>Sharing Tasks</i> ▪ <i>Fraction Language</i> 			<p><i>Van De Walle Pg 213</i></p>
	<p>12.3 Counting Fractional Parts</p>	<p>By counting fractional parts we can help children develop a completely generalized system for naming fractions before they learn about fraction symbolism. Oral names can then be connected to the fraction notation. Ask more than one or less than one?</p>	<p><i>Van De Walle Pg 213</i></p>
	<p><i>15. 3 More Less or Equal to One Whole</i></p>	<p>Give students a collection of fractional parts (all the same type) and indicate the kind of fractional part they have. Physical models can be placed in a plastic baggie with an identifying card, “these are eighths”. The task is to decide if the collection is less than one whole, equal to one whole or more than one whole.</p>	<p><i>Van De Walle pg 248</i></p>
	<p>Comparisons and Equivalencies</p>	<p><i>Gator Pie, Mathews, 1979</i> <i>The Doorbell Rang, Hutchins, 1986</i> <i>The Man Who Counted: A collection of mathematical adventures, Tahan, 1993</i></p>	<p><i>Van De Walle pg 225-226</i></p>
	<p><i>2.4.E</i> Black Holes and Bright Parts</p>	<p>To understand concept of fair shares.</p>	<p><i>Fabulous Fractions, AIMS pg 5-6.</i></p>
	<p><i>2.4.E</i> <i>One Third or Not</i> <i>Thirds and Fourths or Not</i> <i>Two equal pieces show one-half.</i></p>	<p>Use Cuisenaire rods to find equal pieces to form fractions.</p>	<p><i>Everything’s Coming Up Fractions</i> <i>Cuisenaire Rods, pgs 1-6</i></p>

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	2.4.E Pattern Block Relationships	Students explore different wholes using pattern blocks and the fractional parts that cover the whole.	<i>Fractions with Pattern Blocks, pgs. 1-29.</i>
	2.4.E <i>Exploring Fractions with Pattern Blocks</i>	Pattern blocks are used as a way to think about halves, thirds, and sixths through comparing shapes and their areas.	<i>Lessons for Introducing Fractions 4-5, pg 39-45.</i>

Connecting Concepts with Symbolism / Understanding Fraction Symbols <ul style="list-style-type: none"> Meaning of the Top and Bottom Number The top number counts. The bottom number tells <i>what is being counted.</i> <i>Fraction-Parts Counting</i> Numerator and Denominator			<i>Van De Walle pg 214 or pg. 247-249</i>
	15.5 Calculator Fraction Counting		<i>Van De Walle pg 249.</i>
	2.4.E KU 6 Understanding Fractional Number A fractional number can be written as a division or as a decimal.	Students who have achieved this outcome at Level 4 link division and fractions and comfortably interchange $2 \div 3$ with $1/3$ of 2 and $2/3$. They can also change between fractions and decimals where the equivalences are easily visualized or drawn (0.2 is one fifth).	<i>First Steps in Mathematics: Number, Volume 1, pg 144-145.</i>
	2.4.E Sharing (KU 6)	Students solve practical sharing problems in which a smaller number of objects is shared among a larger number of people.	<i>First Steps in Mathematics: Number, Volume 1, pg 146</i>
	2.4.E & 3.3.A <i>Working with Common Fractions and Exploring Equivalence</i> Session 4.1 Reading and Writing Common Fractions	Students use an area model to review reading and writing fractions.	<i>ORIGO, Grade 4</i>
	2.4.E Session 4.5 Halves and Fourths of Collections	Students use base-10 blocks to find halves and fourths of collections.	<i>ORIGO, Grade 4</i>

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	Black Holes and Bright Parts	To recognize fractional numbers as equal parts of a whole	<i>Fabulous Fractions, AIMS pg 5-6.</i>
<ul style="list-style-type: none"> Mixed Numbers and Improper Fractions 			<i>Van De Walle</i>
	12.4 Mixed Number Names	Finding a single fraction that names the same amount as a mixed number (such as $3\frac{1}{2}$) and starting with a fraction greater than 1 finding the mixed number.	<i>Van De Walle pg 215</i>
	<i>15.4 Mixed Number Names</i>	Give students a mixed number such as $3\frac{2}{5}$. Their task is to find a single fraction that names the same amount. They must be able to give an explanation for their result. Have students start with a fraction greater than 1, such as $1\frac{7}{4}$, and have them determine the mixed number and justify their results.	<i>Van De Walle Pg. 249</i>
	12.5 Calculator Fraction Counting		<i>Van De Walle pg 215</i>
	Figures 12.7, 12.8, 12.9	Two or three challenging parts- and whole questions can make an excellent lesson Circle models are not used for these lessons.	<i>Van De Walle pg 215-216</i>
	<i>2.4.E</i> Session 6: Counting and Equivalencies	Involve students in counting fractions beyond one.	<i>Making Sense of Fractions, pg 25.</i>
<ul style="list-style-type: none"> Exercises with Parts and Wholes 			<i>Van De Walle pg 217 or pg 250</i>
	Given the part and the fraction, find the whole (Fig. 12.8) Given the whole and the part, find the fraction (Fig. 12.9) <i>Parts and Wholes Figure 15.9, 15.10, 15.11</i>	Put fractions in order from least to most.	<i>Van De Walle pg 217 or pg 250</i>

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Fraction Number Sense/Fraction Number Sense Flexibility with Fractional Parts			Van De Walle
3.3.A Represent fractions that have denominators of 2, 3, 4, 5, 6, 8, 9, 10, 12 as parts of a whole, parts of a set, and points on the number line			WA State Math Standards
The focus is on numbers less than or equal to one. Students should be familiar with using words, pictures, physical objects and equations to represent fractions			
	12.6 Ordering Unit Fractions	List a set of unit fractions such as $\frac{1}{3}$, $\frac{1}{8}$, $\frac{1}{5}$, and $\frac{1}{10}$. Ask children to put the fractions in order from least to most. The first few times have them explain ideas by using models.	Van De Walle pg. 217
	12.7 Taking Fractions Apart	Assign a fraction to work with. Consider $1\frac{1}{4}$, the task is to find ways to write the fraction in terms of two parts.	Van De Walle pg. 217
	3.3.A & 3.3.D Partitioning Paper (KU 1)	Students find ways to partition a strip of paper into two equal lengths and explain why each part is called a half. Then halve a half to make fourths, etc.	First Steps in Mathematics: Number, Volume 1, pg 96.
	3.3.A & 3.3.D Collections (KU 1)	Extend Partitioning Paper by using different collections.	First Steps in Mathematics: Number, Volume 1, pg 96.
	3.3.A & 3.3.D Halving Grids (KU 1)	Students color half the squares on 4 x 4 grids to show different representations of a half.	First Steps in Mathematics: Number, Volume 1, pg 96.
	3.3.A & 3.3.D Sorting Shapes (KU 1)	Students sort shapes according to whether they have half the area shaded or not.	First Steps in Mathematics: Number, Volume 1, pg 96.
	3.3.A Halves, Quarters, Eighths (KU 1)	Students fold a paper circle in half and label. Fold in half and label.	First Steps in Mathematics: Number, Volume 1, pg 99.
	3.3.A Halving Patterns (KU 1)	Students use a long strip of paper to explore halving and re-halving, naming the new fractional parts each time.	First Steps in Mathematics: Number, Volume 1, pg 99.

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	3.3.A Halving Unit Fractions (KU 1)	Students use strips of paper to investigate finding a half of unit fractions.	<i>First Steps in Mathematics: Number, Volume 1, pg 99.</i>
	3.3.A Representing Half (KU 1) A good integration with LA perhaps for 'Half Day'!	Students make a 'Half' book to give to Kindergarten or 1 st grade students showing all the different ways that a half might be represented.	<i>First Steps in Mathematics: Number, Volume 1, pg 100.</i>
	3.3.A & 3.3.D Sharing Halves (KU 1)	Students explore unusual ways of sharing different items into halves. They explain the context in which their halving might make sense.	<i>First Steps in Mathematics: Number, Volume 1, pg 100.</i>
	3.3.A Sharing a Pizza (KU 2)	Students cut up a variety of different-sized paper circle "pizzas" into different numbers of equal shares.	<i>First Steps in Mathematics: Number, Volume 1, pg 108.</i>
	3.3.A Three Colors (KU 2)	Students shade a 3 x 3 grid with three colors, making sure that each color covers an equal area.	<i>First Steps in Mathematics: Number, Volume 1, pg 108.</i>
	3.3.A Equal Portions (KU 2)	Students use counters to decide how many different ways a packet of 24 sweets could be split into equal portions.	<i>First Steps in Mathematics: Number, Volume 1, pg 108.</i>
	3.3.A Equal Groupings (KU 2)	Variation of "Equal Portions" activity.	<i>First Steps in Mathematics: Number, Volume 1, pg 108.</i>
	2.4.E & 3.3.A Chocolate Bars (KU 2)	Students use diagrams to decide how to share any number of chocolate bars between three people.	<i>First Steps in Mathematics: Number, Volume 1, pg 108.</i>
	3.3.A & 3.3.D Orange Juice (KU 2)	Students investigate the results of sharing a container of orange juice holding 5 cupfuls.	<i>First Steps in Mathematics: Number, Volume 1, pg 109.</i>
	3.3.A & 3.3.D Sharing Large Collections (KU 2)	Students use a range of methods to share large collections into equal parts.	<i>First Steps in Mathematics: Number, Volume 1, pg 109.</i>

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	3.3.A & 3.3.D Pets (KU 2)	Students share different types of materials into the same number of equal parts, i.e. Share food and water equally between a number of pets.	<i>First Steps in Mathematics: Number, Volume 1, pg 109.</i>
	3.3.A & 3.3.D Pattern Blocks (KU 2)	Students use geometrically designed materials such as pattern blocks to partition the larger shapes into equal parts.	<i>First Steps in Mathematics: Number, Volume 1, pg 109.</i>
	3.3.A, 3.3.C & 3.3.D Multiple Slices (KU 2)	Students investigate different sharing contexts using materials or diagrams to partition things so that they can be shared equally by groups of different sizes.	<i>First Steps in Mathematics: Number, Volume 1, pg 109.</i>
	3.3.A & 3.3.D Patterns (KU 2)	Students work in pairs to design tile patterns using equal quantities of three colors.	<i>First Steps in Mathematics: Number, Volume 1, pg 110.</i>
	3.3.A & 3.3.D Rations (KU 2)	Students partition a range of materials or create diagrams in order to decide how much each person would receive in a given situation.	<i>First Steps in Mathematics: Number, Volume 1, pg 110.</i>
	3.3.A & 3.3.D Rectangles (KU 2)	Students make a list of rectangles that can be made from a specified number of tiles.	<i>First Steps in Mathematics: Number, Volume 1, pg 110.</i>
	3.3.A & 3.3.D Splitting a Circle (KU 2)	Students use a protractor to split a circle into equal parts, then use the divisions for an art activity. Students decide and demonstrate what fraction of a circle they used in their picture.	<i>First Steps in Mathematics: Number, Volume 1, pg 110.</i>
	3.3.A & 3.3.D Fraction Sequence (KU 2)	Students use a protractor or circular grid to divide a circle into thirds, then one-third into thirds.	<i>First Steps in Mathematics: Number, Volume 1, pg 110.</i>
	3.3.A, 3.3.D & 4.3.C Dividing Land (KU 2)	Students use a 10 x 10 grid to represent farm land. Students divide the land into five paddocks of equal area.	<i>First Steps in Mathematics: Number, Volume 1, pg 110.</i>
	3.3.A Fractions and Shapes (KU 3)	Students use pattern blocks, beginning with the hexagon as a whole. They decide what fraction of a hexagon the other shapes represent.	<i>First Steps in Mathematics: Number, Volume 1, pg 121.</i>

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	3.3.A Directions (KU 3)	Students use 'quarter turn', 'half turn', 'three quarter' turn, 'right' and 'left' to give directions to travel around the classroom.	<i>First Steps in Mathematics: Number, Volume 1, pg 121.</i>
	3.3.A Sharing Sweets (KU 3)	Show the class 20 sweets. Ask: If I ate five sweets, what fraction of the sweets have I eaten?	<i>First Steps in Mathematics: Number, Volume 1, pg 122.</i>
	3.3.A Fraction Machine (KU 3)	Extend Sample Learning Activity "Function Box (KU 1, pg. 95) by asking the machine to make four-fifths of different collections and/or objects.	<i>First Steps in Mathematics: Number, Volume 1, pg 122.</i>
	3.3.A Measuring Fractions (KU 3)	Students use measuring tools (scales, rulers, grid paper, and measuring jugs) to find a fraction of a range of 'wholes'. Students record their work in words, diagrams and numerals.	<i>First Steps in Mathematics: Number, Volume 1, pg 123.</i>
	3.3.A Fractions and Shapes (KU 3)	Students use pattern blocks, beginning with a trapezium as the whole.	<i>First Steps in Mathematics: Number, Volume 1, pg 123.</i>
	3.3.A Finding Wholes (KU 3)	Students use pattern blocks to find the whole shape when a triangle represents one third (quarter, sixth, twelfth). Students record using fraction symbols.	<i>First Steps in Mathematics: Number, Volume 1, pg 123.</i>
	3.3.A Fractional Values (KU 3)	Students use pattern blocks to make a pattern. This becomes the whole unit. Ask students to find the fractional value of each piece as it relates to the whole.	<i>First Steps in Mathematics: Number, Volume 1, pg 123.</i>
	3.3.A Stars (KU 3)	Students show and explain how a fraction, such as two-thirds, means two out of every three.	<i>First Steps in Mathematics: Number, Volume 1, pg 124.</i>
	3.3.A Parts of a Whole (KU 3)	Students are asked to find parts of a whole where the whole itself is part of something else.	<i>First Steps in Mathematics: Number, Volume 1, pg 124.</i>
	3.3.A Three Quarters (KU 3)	Students sort illustrations into two categories – those that can be represented by three quarters and those that can't.	<i>First Steps in Mathematics: Number, Volume 1, pg 124.</i>

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	2.4.D & 3.3.A Thirds (KU 3)	Students are asked to group several different-sized collections into three equal groups. Students are asked, ‘Why don’t they have the same quantity?’ “	<i>First Steps in Mathematics: Number, Volume 1, pg 125.</i>
	3.3.A, 3.3.B, 3.3.D KU 5 Fractional Number We can compare and order fractional numbers and place them on a number line.	<ul style="list-style-type: none"> ▪ Compare each fraction to a ‘benchmark’ number (often 1 or $\frac{1}{2}$) ▪ Think about each fraction’s distance from 1 ▪ Find a whole number that each fraction will divide and compare how many you will get 	<i>First Steps in Mathematics: Number, Volume 1, pg 134-135.</i>
	3.3.A Ropes (KU 5)	Stretch a skipping rope across the classroom floor or wall. Mark one end ‘0’ and the other end ‘1’. Invite students to stand on or next to the rope to indicate fractional parts.	<i>First Steps in Mathematics: Number, Volume 1, pg 136.</i>
	3.3.A Comparing Halves (KU 5)	Provide students with two obviously different sized wholes, each split into halves. Discuss the difference between halves.	<i>First Steps in Mathematics: Number, Volume 1, pg 136.</i>
	3.3.A & 3.3.B Fraction Tapes (KU 5)	Students see how fractions fit with whole numbers. First have them fold identical lengths of paper tape into various fractional parts. Ask students to label the folds and then the ends. Draw out the idea that fractions on a tape show a position on the tape.	<i>First Steps in Mathematics: Number, Volume 1, pg 137.</i>
	3.3.A & 3.3.B Finding Fractions (KU 5)	Ask students to fold a paper strip to find a given fraction. Give students different-sized strips of paper. Ask students to find someone else in the room with the same sized strip and compare fractions.	<i>First Steps in Mathematics: Number, Volume 1, pg 137.</i>
	3.3.A & 3.3.B Estimating Positions (KU 5) (extension of Finding Fractions)	Gives students several strips of paper the same size. Ask them to estimate without folding, the position of a half, third, quarter, three-quarters and two-thirds.	<i>First Steps in Mathematics: Number, Volume 1, pg 138.</i>
	3.3.A & 3.3.B Sharing Chocolate (KU 5)	Pose this problem to students: I was offered the choice of half, a quarter or a third of a chocolate bar. Which one would have given me the most chocolate?	<i>First Steps in Mathematics: Number, Volume 1, pg 138.</i>

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	<p>3.3.A Comparing Lengths (KU 5)</p>	<p>Create a number line with string. Add cards labeled '0' at one end and '1' at the other. Ask students to determine where fraction cards would be positioned on the line and justify their suggestions. Draw out the idea that there is a much greater difference between $\frac{3}{4}$ and $\frac{4}{4}$ than there is between $\frac{11}{12}$ and $\frac{12}{12}$.</p>	<p><i>First Steps in Mathematics: Number, Volume 1, pg 139.</i></p>
	<p>3.3.A <i>Working with Division Session 4.4 Relating Fractions to Division: Halves and Quarters.(1 session)</i></p>	<p>Find one-half or one-quarter of a collection and relate the process to division.</p>	<p><i>ORIGO Grade 3</i></p>
	<p>3.3.A Investigation 1: Session 3-4 <i>More Brownies to Share</i> (2 hrs. min.)</p>	<p>Fractional parts of a whole. Fractional parts must be equal size, but can be a different shape. Conventional fraction notation. Becoming familiar with grouping unit fractions, $\frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{3}{6}$</p>	<p><i>Investigations: Fair Shares 3rd Pg. 13-21.</i></p>
	<p>3.3.A <i>Pattern Block Cookies, Investigation 2: Session 1-2 Cookie Shares</i> (min. 2 hrs.)</p>	<p>Finding shapes to cover one whole, identifying fractional parts that add to one whole, and writing fractional parts.</p>	<p><i>Investigations: Fair Shares 3rd Pg.24</i></p>
	<p>3.3.A & 6.2.C <i>Investigation 1: Exploring Percents and Fractions;</i> <i>Session 2:Percent Grid Patterns</i></p>	<p>Students use fractions to describe how many in a group share a particular characteristic while partitioning a whole.</p>	<p><i>Investigations: Name that Portion 5th, pg 10</i></p>
	<p>3.3.A <i>Fraction Models</i> (five lessons)</p>	<p>Students explore fractions using area, length and set models.</p>	<p><i>Navigating Through Number and Operations 3-5, pg. 27</i></p>
	<p>3.3.A & 3.3.D <i>Balloons and Brownies</i> (two class periods; lesson can be repeated multiple times)</p>	<p>Students experience thinking about fractions as parts of wholes and as parts of sets. The contexts of balloons and brownies present students with situations where objects can and cannot be divided into smaller parts.</p>	<p><i>Lessons for Extending Fractions, Grade 5, pg 27-38.</i></p>

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	3.3.A & 3.3.D <i>Fraction Scavenger Hunt</i>	Illustrate in multiple ways how a given fraction can be represented as a part of a group or set of objects. Heightened awareness of fractions in real world.	<i>Fabulous Fractions, AIMS pg 4</i>
	2.4.E, 3.3.B & 3.3.D <i>Investigation 1: Using Fractions to Describe Data; Session 1: Guess My Rule; Session 2: Finding Familiar Fractions</i>	Partitioning a group according to a rule Finding familiar Fractions of a group Estimating complex fractions with familiar fractions	<i>Investigations: Three out of Four Like Spaghetti, 4th, pg. 2-16.</i>
	3.3.B & 3.3.D <i>Investigation 2: Looking at Data in Categories Session 5, 6, & 7: Making Comparisons with all the Data (min. 3 hrs.)</i>	Using fractions to compare categorical data from two groups	<i>Investigations: Three out of Four Like Spaghetti, 4th, pg. 45.</i>
	3.3.A & 3.3.B <i>The Fraction Kit (at least 5 class periods)</i>	Students cut and label fractional parts of paper strips. There are several games to play that connect their experience to representing and comparing fractions.	<i>Lessons for Introducing Fractions 4-5, pg 10-29.</i>
	3.3.A Sharing (KU 6)	Students solve practical sharing problems in which a smaller number of objects is shared among a larger number of people.	<i>First Steps in Mathematics: Number, Volume 1, pg 146</i>
	3.3.A & 3.3.D Sharing Paper (KU 6)	Students work in groups to share strips of paper equally.	<i>First Steps in Mathematics: Number, Volume 1, pg 147</i>
	3.3.A, 3.3.D, 4.2.F, 4.2.I Chocolate Bars (KU 6)	Repeat practice predicting and testing sharing a collection of pretend chocolate bars between various numbers of people until students are able to write their own rule for working out what fraction of bar each person will get.	<i>First Steps in Mathematics: Number, Volume 1, pg 147.</i>

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Benchmarks of Zero, One-Half, or One			Van De Walle
3.3.B Compare and order fractions that have denominators of 2, 3, 4, 5, 6, 8, 9, 10, 12.			Mathematics Standards
	12.8 (15.6) Zero, One-half, or One	Sort fractions into three groups – close to 0, close to $\frac{1}{2}$ and close to 1	Van De Walle pg 218 (251)
	12.9 (15.7) Close Fractions	Name a fraction close to 1. Then keep naming fractions closer to 1 and explain why it is closer	Van De Walle pg 218 pg 251
	12.10 (15.8) About How Much?	Estimating fractional parts	Van De Walle pg 218 (252)
	3.3.B Working w/Common Fractions & Exploring Equiv. Fractions Session 4.2 Investigating Relative Position	Students use the number line to identify which reference point (zero, one-half or one) is closest	ORIGO, Grade 4
	3.3.A What Number Am I? (KU 5)	Pose this problem to students: I am less than one but more than zero. I am bigger than one half. Students guess the number and then discuss strategies they used to work out the answer.	First Steps in Mathematics: Number, Volume 1, pg 137.
	3.3.B & 3.3.C Actions on Fractions (one to three days)	Students expand understanding of processes for comparing and ordering fractions. Students compare using concrete models, benchmarks and order fractions using concrete, benchmark and parallel number lines.	Navigating Through Number and Operations 3-5, pg 34.
	3.3.A & 3.3.C Fractions as Parts of Sets (at least one class period)	Lesson focuses on using fractional notation to represent parts of groups and comparing fractions to one-half.	Lessons for Introducing Fractions 4-5, pg 1-9.
	3.3.B Introducing One-half as a Benchmark (15-20 min. to be repeated)	The lesson suggests a way to introduce one-half as a benchmark and then provide students practice using $\frac{1}{2}$ to compare fractions.	Lessons for Introducing Fractions 4-5, pg 54-61.

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	<p>3.3.B <i>How Much More to Make One-half?</i> (two half hour sessions)</p>	Students figure out how much more is needed to make $\frac{1}{2}$ when given various fractions. Emphasis is kept on student's thinking and reasoning.	<i>Lessons for Introducing Fractions 4-5, pg 68-74.</i>
	<p>3.3.B <i>Fractions and Venn Diagrams</i> (two class periods, + additional 15 min periods)</p>	Uses Venn diagrams to provide students experience with comparing fractions.	<i>Lessons for Extending Fractions, Grade 5, pg 39-52</i>
	<p>3.3.B <i>Fractions in Contexts</i> (one class period)</p>	Students develop understanding of the concept of one-half. This lesson presents various situations and asks students to determine if it is "exactly $\frac{1}{2}$ " "about $\frac{1}{2}$ " "less than $\frac{1}{2}$ " or "more than $\frac{1}{2}$ ".	<i>Lessons for Introducing Fractions 4-5, pg 30-39.</i>
<p>Thinking About Which is More</p> <ul style="list-style-type: none"> ▪ Concepts not rules ▪ Conceptual Thought Patterns for Comparison 			<i>Van De Walle</i>
<p>3.3.B Compare and order fractions that have denominators of 2, 3, 4, 5, 6, 8, 9, 10, 12.</p>			<i>Mathematics Standards</i>
	Figure 12.11	Comparing fractions using concepts	Van De Walle pg 219
	<i>15.9 Ordering Unit Fractions</i>	List a set of fractions such as $\frac{1}{3}$, $\frac{1}{8}$, $\frac{1}{5}$, and $\frac{1}{10}$. Ask children to put the fractions in order from least to most. Challenge children to defend the way they ordered the fractions. The first few times have them explain their ideas by using models.	<i>Van De Walle pg 252</i>
	<i>15.10 Choose, Explain, Test</i>	Present two or three pairs of fractions to students. The task is to decide which fraction is greater, to explain why they think this is so, and then to test their choice using any model that they wish. Students write explanations.	<i>Van De Walle pg 253</i>
	12.11 Compare and Test	<ul style="list-style-type: none"> ▪ Use a familiar fraction model. ▪ Present a pair of fractions ▪ Students think about and write before using the physical model to compare 	<i>Van De Walle pg 219</i>

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	12.12 Why is it more?	Give a pair of fractions to compare Write as many good explanations as you can to verify your comparison	<i>Van De Walle</i> pg 219
	12.13(15.11) <i>Line 'Em Up</i>	Select four or five fractions for students to put in order from least to most. Have them indicate approx. where each fraction belongs on a number line labeled only with 0, $\frac{1}{2}$, and 1	<i>Van De Walle</i> pg 254
	3.3.B & 3.3.D Comparing Fractions (KU 1)	Students cut and rearrange parts of shapes to compare fractions that look different.	<i>First Steps in Mathematics: Number, Volume 1, pg 97.</i>
	3.3.A & 3.3.B <i>Investigation 1: Exploring Percents and Fractions</i> <i>Sessions 3 & 4: Fraction and Percent Problems</i>	Students name portions with fractions (ignore percents), compare fractions using $\frac{1}{2}$ and 1, and build on knowledge of unit fractions with numerators > 1 .	<i>Investigations: Name that Portion</i> <i>Grade 5, pg 29</i>
	3.3.A 8.1 <i>Representing Fractions with Area Models</i>	In this session students figure out methods to divide squares into equal parts.	<i>ORIGO 5th</i>
	3.3.A 8.2 <i>Linear Model</i>	Students figure out methods to divide lengths into equal parts to show fractions	<i>ORIGO 5th</i>
	3.3.A & 3.3.C Black Holes and Bright Part	To recognize and name equivalent fractions To understand the relative size of fractions by direct comparison.	<i>Fabulous Fractions, AIMS pg 5-19.</i>
	3.3.A, 3.3.B, 3.3.C, 3.3.D <i>Early Investigations: Understanding Fractions</i> <i>Session 1- Rainbow Fractions</i> <i>2 Exploring Fractions around us</i> <i>3 The Fraction Box</i> <i>4 Fraction Tools</i> <i>5 Estimating Fractional Parts</i> <i>6 Tearing Fractions</i> <i>7 Fraction Game</i> <i>8 & 9 Fair Shares</i>	Students write what they know about fractions, cut a “cake” into halves, fourths, eighths, & sixteenths, look for examples in the room, explore different fraction tools, make own fraction kit, combine fractions to make a whole, determine if area models are fair shares.	<i>Making Sense of Fractions, 3-6, pg 7-17.</i>

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	<p>3.3.B <i>The Comparing Game</i> <i>(three class periods + additional playing time)</i></p>	Two-person game that provides students practice representing and comparing fractions	<i>Lessons for Extending Fractions, Grade 5, pg 53-60.</i>
	<p>3.3.B <i>Comparisons and Equivalencies</i> <i>Session 1: Rainbow Fractions: Which is More?</i></p>	Students are encouraged to explore relationships among fraction tiles.	<i>Making Sense of Fractions pg 19.</i>
	<p>3.3.B & 3.3.C <i>Session 2: Fraction Tools</i></p>	Students work with a partner to brainstorm and record observations as they compare fractions or show equivalence.	<i>Making Sense of Fractions pg 20.</i>
	<p>3.3.B <i>Session 7: Walking You Home</i></p>	Students order fractions on a clothesline and explain their thinking.	<i>Making Sense of Fractions pg 26.</i>
	<p>3.3.B <i>In Size Order</i> <i>(two class periods)</i></p>	Students experience comparing and ordering fractions. Students generate a set of 8 fractions between 0 and 1. Students order the numbers from smallest to largest.	<i>Lessons for Extending Fractions, Grade 5, pg 14-26.</i>
	<p>3.3.B & 3.3.C <i>Between Zero and One</i></p>	To provide practice recognizing equivalent fractions To practice ordering fractions	<i>Fabulous Fractions, AIMS, pg. 86-93.</i>
	<p>3.3.B Ordering Fractions: Teaching Notes</p>	Ordering fractions by visualizing in your mind. Will take previous experiences with models.	<i>Classroom Activities for Making Sense of Fractions, Ratios & Proportions, NCTM 2002 Yearbook, pg. 11-13.</i>
Estimation			<i>Van De Walle</i>
	<p>12.14 Pick the Best</p>	Flash sums or differences of proper fractions. More than 1, Less than 1? Closer to 0, 1, or 2? Closer to 0, $\frac{1}{2}$, 1, $1\frac{1}{2}$, or 2.	<i>Van De Walle pg 221</i>
	<p>12.15 Speed Estimates</p>	Number your papers	<i>Van De Walle pg 221</i>

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	12.18 (15.14) Dot Paper Equivalencies	Use grid paper or dot paper so that regions can be easily subdivided into many smaller regions. Students draw a model for a whole, shade a portion of it; then determine how many different names for the shaded region they can find.	<i>Van De Walle</i> pg 223 (258)
	12.19 Arrange the Counters, Find the names 15.15 <i>Group the Counters, Find the Names</i>	Set out a specific number of counters in two colors. This set is the whole. Students find as many different names for the different colors by arranging the counters into different subgroups	<i>Van De Walle</i> pg 223 (259)
	15.16 <i>Missing-Number Equivalencies</i>	Give students an equation expressing equivalence between two fractions but with one of the numbers missing.	<i>Van De Walle</i> pg. 259
	3.3.D Comparing Halves (KU 1)	Students compare halves of two different sized wholes and discuss and explain why these halves are not equal.	<i>First Steps in Mathematics: Number, Volume 1, pg 96.</i>
	3.3.D Half? (KU 1)	Give students a paper showing two portions that do not visually look the same. Students use paper tiles and grid paper to decide whether the paper has been divided into halves.	<i>First Steps in Mathematics: Number, Volume 1, pg 97</i>
	3.3.D Measuring Half (KU 1)	Students use ways other than counting to separate a quantity of paperclips into halves.	<i>First Steps in Mathematics: Number, Volume 1, pg 97.</i>
	3.3.D Halving Wholes (KU 1)	Students find different ways to halve a range of wholes.	<i>First Steps in Mathematics: Number, Volume 1, pg 98.</i>
	3.3.D Chocolate Bars (KU 1)	Students look at two different shaped halves of chocolate bars and determine if they are equal.	<i>First Steps in Mathematics: Number, Volume 1, pg 98.</i>
	3.3.D Half Recipes (KU 1)	Ask students to rewrite cake recipes as ‘half recipes’.	<i>First Steps in Mathematics: Number, Volume 1, pg 99.</i>
	3.3.D Finding Half (KU 1)	Student are given a range of 4 x 4 grids that have been partly shaded. Students identify shapes in which exactly half the area has been shaded.	<i>First Steps in Mathematics: Number, Volume 1, pg 99</i>

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	3.3.D Cows in a Paddock (KU 1)	Students investigate different-shaped pieces of paper each marked to show two portions to determine if the cows will get the same amount of grass.	<i>First Steps in Mathematics: Number, Volume 1, pg 100</i>
	3.3.D Half Sayings (KU 1)	Students list situations in which ‘half’ is used colloquially to refer to attributes not usually measured.	<i>First Steps in Mathematics: Number, Volume 1, pg 100.</i>
	3.3.C, 3.3.D & 4.1.B Sharing with Odd Numbers (KU 2) (continuation of Multiple Slices)	Students investigate different sharing contexts using materials or diagrams to partition things so that they can be shared equally by groups of different sizes this time including numbers that do not have a common factor.	<i>First Steps in Mathematics: Number, Volume 1, pg 109.</i>
	3.3.D Rules for Sharing (KU 2)	Students explore different ways of cutting up three pies to share equally among a family of five.	<i>First Steps in Mathematics: Number, Volume 1, pg 110.</i>
	3.3.D Multiple Slices (KU 2)	Students decide how many parts they need to partition things into so that they have shared equally by groups of different sizes.	<i>First Steps in Mathematics: Number, Volume 1, pg 110.</i>
	3.3.C & 3.3.D Common Factors (KU 2)	Students work out the number of portions needed for different groups of grandchildren that have a common factor where one number is not a factor of the other.	<i>First Steps in Mathematics: Number, Volume 1, pg 110.</i>
	3.3.D The Frog and the Flea (KU 5)	Pose this problem to students: A frog and a flea had a jumping contest. Each of the frog’s jumps was one third of a unit long. Each of the flea’s jumps was one quarter unit long. The winner was the one who reached four units in the fewest jumps.	<i>First Steps in Mathematics: Number, Volume 1, pg 137.</i>
	3.3.D Pocket Money (KU 5)	Pose this problem to students. Mary and John each spent a quarter of their pocket money. Is it possible for Mary to have spent more money than John?	<i>First Steps in Mathematics: Number, Volume 1, pg 137.</i>
	3.3.D Cheesecake (KU 5)	Students think about the size of fractions to solve word problems.	<i>First Steps in Mathematics: Number, Volume 1, pg 138.</i>

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	<p>3.3.C <i>Pattern Block Cookies, Investigation 2: Session 4 Fraction Cookie Game (min. 1 hr)</i></p>	<p>Identify fraction parts, exchange equivalent fractions, and add and subtract fractions using physical models to make exchanges.</p>	<p><i>Investigations: Fair Shares 3rd Pg 34</i></p>
	<p>3.3.D <i>Pattern Block Cookies, Investigation 2: Session 7: Half Yellow</i></p>	<p>Creating a design that is half yellow or another fraction. Recognizing that a design is half or another fraction.</p>	<p><i>Investigations: Fair Shares 3rd P 40</i></p>
	<p>3.3.B & 3.3.C <i>Investigation 1: Parts of Squares: Halves, Fourths, and Eights Session 1: Finding Halves of Crazy Cakes (min. 1 hr.)</i></p>	<p>Understanding that equal fractions of a whole have the same area Understanding that cutting and pasting shapes conserves their area. Find halves of shapes.</p>	<p><i>Investigations: Different Shapes, Equal Pieces, 4th pg. 4-7.</i></p>
	<p>3.3.A & 3.3.D <i>Investigation 1: Sessions 2, 3, 4: Halves, Fourths, & Eighths w/geoboards (min. 3 hrs.)</i></p>	<p>$\frac{1}{2}$, $\frac{1}{4}$ and $\frac{1}{8}$ represented on Geoboards and dot paper. Focuses on equal pieces that are not necessarily congruent.</p>	<p><i>Investigations: Different Shapes, Equal Pieces, 4th pg 8</i></p>
	<p>3.3.C <i>Investigation 2: Session 1 & 2: Thirds, Sixths, and Twelfths (min. 2 hrs)</i></p>	<p>Using geoboards if needed and dot paper to record solutions, students divide area models into $\frac{1}{3}$s, $\frac{1}{6}$s, and $\frac{1}{12}$ths of different wholes. The pieces have equal area but are not necessarily congruent.</p>	<p><i>Investigations: Different Shapes, Equal Pieces 4th, pg 24.</i></p>
	<p>3.3.C Investigation 2: <i>Session 4: Working with $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{7}{8}$ (min. 1 hr.)</i></p>	<p>Comparing fractions (bigger than, smaller than, equivalent) $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{8}$</p>	<p><i>Investigations: Different Shapes, Equal Pieces 4th, pg 33</i></p>
	<p>3.3.C <i>Investigation 3: Session 4 & 5: Making a Fraction Number Line (2 hrs. min.)</i></p>	<p>Comparing the relative sizes of common fractions – unit fractions, same denominator, same numerator. Comparing any fraction with $\frac{1}{2}$ and 1. Comparing fractions that are missing one part of the whole.</p>	<p><i>Investigations: Different Shapes, Equal Pieces 4th, pg 49.</i></p>
	<p>3.3.C <i>Session 4.3 Exploring Equivalent Fractions</i></p>	<p>Students use concrete and picture materials to find different ways to describe the same part of one whole.</p>	<p><i>ORIGO, Grade 4</i></p>

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	<p>3.3.C <i>Session 4.4 Writing Equivalent Fractions</i></p>	Students used a fraction mat to find and then write equivalent fractions.	<i>ORIGO, Grade 4</i>
	<p>3.3.C <i>Same Name Frame</i></p>	To construct a model for generating equivalent fractions To promote meaning in building a table of fraction equivalents connected to the multiplication table and multiple names of ONE.	<i>Fabulous Fractions, AIMS, PG. 43-45.</i>
	<p>3.3.C <i>Dividing Brownies</i> <i>(four class periods + four additional days for the extension)</i></p>	Students explore different ways to divide “brownies” into halves, fourths, and eighths. Students can divide brownies either by using spatial approach or a numerical approach.	<i>Lessons for Extending Fractions 5, pg 1-13.</i>
	<p>3.3.C <i>Session 3: Fraction Game: What you spin is what you take</i></p>	Students will develop their concept of equivalence through trades for the same fraction color	<i>Making Sense of Fractions pg 21.</i>
	<p>3.3.C <i>Fair Pairs for One and All</i></p>	To provide practice recognizing equivalent fractions To provide practice recognizing part/whole relationships.	<i>Fabulous Fractions, AIMS, pg. 94.</i>
	<p>3.3.C <i>Folding Fractions</i></p>	Copy	<i>AIMS Magazine Fall 2007, pg 28</i>
	<p>3.3.C <i>Session 4: Fraction Game: Which is Less? Which is More?</i></p>	Partner game where students compare fractions.	<i>Making Sense of Fractions pg 23.</i>
	<p>3.3.C <i>Drawing Fractional Parts of Sets</i> <i>(one to two class periods)</i></p>	Students practice representing fractions by making sketches. The lesson divides circles, squares, and rectangles and provides experience with equivalent fractions.	<i>Lessons for Introducing Fractions 4-5, pg 46-53.</i>
	<p>3.3.C <i>When is a Fraction Worth One-Half?</i> <i>(one class period)</i></p>	Students are asked to come up with as many different ways as they can to explain why a fraction is equivalent to one-half. Algebraic thinking is translated to generalization using variables to represent numerators and denominators.	<i>Lessons for Introducing Fractions 4-5, pg 62-67.</i>

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	3.3.D <i>Comparison Problems</i>	Provide a rich variety of problem for comparison of fractions.	<i>Making Sense of Fractions pg 24.</i>
	3.3.D <i>Fraction Sequences</i> <i>(one class period)</i>	Students consider this question: If you add one to the numerator and denominator of a fraction, is the new fraction always greater than the original fraction? Students practice with comparing and ordering fractions.	<i>Lessons for Extending Fractions, Grade 5, pg 75-81</i>
	3.3.D <i>Session 8: Fraction Riddles and Problems</i>	Pose problems and riddles for students to solve. Challenge students to find as many solutions as possible or to create their own riddles for others to solve.	<i>Making Sense of Fractions pg 27.</i>
	3.3.C Fraction Fringe on the Cutting Edge	To understand relative size of fractions by direct comparison using a linear model To recognize and name fractions as fair shares of a linear unit. To construct a linear model that represents fractions and proportionality.	<i>Fabulous Fractions, AIMS, pg. 37-42.</i>
	3.3.C Jelly Bean Rounds	To recognize and name fractions as representative of a counted part of a group or set of objects To understand the meaning of a numerator and denominator in the symbolic form of a fraction.	<i>Fabulous Fractions, AIMS, pg. 82-85.</i>
	3.3.C Rename It: Teaching Notes	Equivalent fractions	<i>Classroom Activities for Making Sense of Fractions, Ratios & Proportions, NCTM 2002 Yearbook pg14-16</i>
	3.3.D & 4.2.I Chocolate Bars (KU 6)	Students Use pretend chocolate bars made from rectangles of paper to model sharing two bars among three people. Activity is repeated for different numbers of bars and different numbers of people. The link between division and fractions is made.	<i>First Steps in Mathematics: Number, Volume 1, pg 149.</i>

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<p>Developing an Equivalent Fraction Algorithm Uses pictorial bridge to developing algorithm</p> <ul style="list-style-type: none"> ▪ An Area Model ▪ Writing Fractions in Simplest Terms ▪ Multiplying by One 	<p><i>Van De Walle</i></p>		
<p>End of Factoring Phase By the end of the Factoring phase, students can:</p> <ul style="list-style-type: none"> ▪ Visualize an array to see that five blue counters is one third of a bag of 15 counters; ▪ Visualize or draw their own diagrams to compare fractions with the same denominator or simple equivalences ▪ The idea of splitting a whole into parts to understand decimals and fraction equivalents; <p>Relate fractions and division.</p>	<p><i>First Steps: Diagnostic Map - Number</i></p>		
<p>4.2.A Represent decimals through hundredths with place value models, fraction equivalents and the number line. 4.2.B Read, write, compare, and order decimals through hundredths. 4.2.C Convert a mixed number to a fraction and vice versa, and visually represent the number. 4.2.D Convert a decimal to a fraction and vice versa, and visually represent the number. 4.2.E Compare and order decimals and fractions (including mixed numbers) on the number line, lists, and the symbols $<$, $>$, or $=$. 4.2.F Write a fraction equivalent to a given fraction. 4.2.G Simplify fractions using common factors. 4.2.H Round fractions and decimals to the nearest whole number. 4.2.I Solve single-and multi-step word problems involving comparison of decimals and fractions (including mixed numbers), and verify the solutions.</p>	<p><i>Mathematics Standards</i></p> <p><i>Mathematics Standards</i></p>		
	<p>12.20 (15.17) Slicing Squares</p>	<p>Begin with simple contextual tasks Connect the meaning of fraction computation with whole-number computation Let estimation and informal methods play a big role in the development of strategies Explore each of the operations using models</p>	<p><i>Van De Walle pg 224 (260)</i></p>
	<p>4.2.E & 4.2.I <i>Investigation 2: Pattern Block Cookies, Session 3: Comparing Shares</i></p>	<p>Working with fractions and mixed numbers, determining fair shares, comparing fractions, proving which fraction is larger</p>	<p><i>Investigations: Fair Shares 3rd Pg 30</i></p>
	<p>4.2.I <i>Investigation 2: Pattern Block Cookies,</i></p>	<p>Putting pieces together to make wholes while working with mixed numbers. Looking for number patterns.</p>	<p><i>Investigations: Fair Shares 3rd Pg 38</i></p>

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	<i>Session 5-6 Backward Sharing, (3 hours min)</i>		
	4.2.C <i>Investigation 3: Other Things to Share Session 1 & 2: How Can We Split Balloons? (2 hour min.)</i>	Understanding the relationship between fractions and division. Relating notation for common fractions with notation for decimals on the calculator.	<i>Investigations: Fair Shares, 3rd grade, pg 46-55.</i>
	4.2.C <i>Investigation 3: Other Things to Share Session 3: Sharing Many Things (min. 1 hr.)</i>	Understanding the relationship between fractions and division. Relating notation for common fractions with notation for decimals on the calculator.	<i>Investigations: Fair Shares, 3rd grade, pg 56-59.</i>
	4.2.C <i>Investigation 3: Session 1 & 2: Making Fraction Cards (2 hrs. min.)</i>	Making fraction cards including improper fractions and mixed numbers. Play Fraction Fish.	<i>Investigations: Different Shapes, Equal Pieces 4th, pg 40.</i>
	4.2.E <i>Investigation 3: Session 3: Ordering fractions with respect to benchmarks. (min. 1 hr.)</i>	Comparing fractions with the landmarks of 0, $\frac{1}{2}$, 1 and 2. Discriminating among fractions less than and greater than 1.	<i>Investigations: Different Shapes, Equal Pieces 4th, pg 46.</i>
	4.2.B & 4.2.E <i>Exploring Percents and Fractions Session 1: Connecting Fractions, Decimals and Percents (6.3.C)</i>	Using fractions to describe how many in a group share a particular characteristic Representing fractions and percents on a number line Finding equivalent fractions and percents Using $\frac{1}{2}$ and 1 as references in ordering fractions and percents	<i>Investigations: Name that Portion Grade 5, pg 2-9.</i>
	4.2.A <i>Investigation 1: Exploring Percents and Fractions Sessions 3 & 4: Fraction and Percent Grids</i>	Representing common fractions on 10 by 10 grids. Identifying equivalent fractions and percents. <i>Could this be changed to decimals?</i>	<i>Investigations: Name that Portion Grade 5, pg 18-23</i>
	4.2.A Fractions of a Meter (KU 3)	Students are invited to explore fractional relationships within a square meter.	<i>First Steps in Mathematics: Number, Volume 1, pg 125.</i>

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	4.2.E Measuring Meters (KU 3)	Each member of a group of 3 folds a meter long paper strip into fourths and then join the strips to make a 3 meter tape.	<i>First Steps in Mathematics: Number, Volume 1, pg 120.</i>
	4.2.E Thirds (KU 3)	Extension of 'Measuring Meters' activity where students fold the 3 meter tape into thirds.	<i>First Steps in Mathematics: Number, Volume 1, pg 121.</i>
	4.2.E Fraction Words (KU 3)	Students draw their interpretations of fraction words used in contest.	<i>First Steps in Mathematics: Number, Volume 1, pg 122.</i>
	2.4.E & 4.2.E Fraction Number Line (KU 5)	Draw a number line on the ground or on a large sheet of paper with units and half units marked. Have students jump in units, half units and/or quarter units, counting as they go.	<i>First Steps in Mathematics: Number, Volume 1, pg 138.</i>
	4.2.F Equal Parts (KU 3)	Students are challenged to think about fraction equivalences as they ply with models partitioned into equal parts.	<i>First Steps in Mathematics: Number, Volume 1, pg 128.</i>
	4.2.F Comparing Fractions (KU 3)	Students are asked to fold one strip of paper into halves, another into quarters and another into eighths. Students compare the strips	<i>First Steps in Mathematics: Number, Volume 1, pg 128.</i>
	4.2.F Fraction Circles (KU 3)	Students are asked to compare fractions using three paper circles that are equal in size. One is cut into halves, another into quarters and the third into eighths.	<i>First Steps in Mathematics: Number, Volume 1, pg 128.</i>
	4.2.F Halves and Quarters (KU 3)	Students compare two sandwiches of equal size, but cut into different fractional pieces. Are two quarters the same amount as one half?	<i>First Steps in Mathematics: Number, Volume 1, pg 128.</i>
	4.2.F Fractions and Collections (KU 3)	Students compare the number of sweets in a half share of one collection of 8 sweets with the number of sweets in two quarter shares of another collection of 8 sweets.	<i>First Steps in Mathematics: Number, Volume 1, pg 128.</i>
	4.2.F Representing Fractions (KU 3)	Students use identical paper sheets to make a chart that show some of the ways any given fractional amount could be represented.	<i>First Steps in Mathematics: Number, Volume 1, pg 129.</i>
	4.2.F Equivalent Fractions (KU 3)	Students use materials (strips of paper, fraction cakes, pattern blocks) to find as many different fractions as they can that are equivalent to one-half.	<i>First Steps in Mathematics: Number, Volume 1, pg 129.</i>

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	4.2.F Fraction Circles (KU 3)	Students compare fractions using four paper circles that are equal in size. Students explore equivalent fractions by matching sections of the circles.	<i>First Steps in Mathematics: Number, Volume 1, pg 129.</i>
	4.2.F Equivalence (KU 3)	Students are asked to consider this situation: Andrew says that ‘three quarters equals six eighths!’ Angela says, ‘Not always.’ Students are asked to explore equivalence and explain how both students can be right.	<i>First Steps in Mathematics: Number, Volume 1, pg 129.</i>
	4.2.F & 4.2.I Chocolate Bars (KU 3)	Students use grid paper representations of chocolate bars to investigate questions.	<i>First Steps in Mathematics: Number, Volume 1, pg 129.</i>
	4.2.F Marbles (KU 3)	Students find equivalent fractions of collections.	<i>First Steps in Mathematics: Number, Volume 1, pg 130.</i>
	4.2.F Equivalent Fractions (KU 3)	Students are asked to fold paper strips, which are equal in length into halves, quarters, sixths, eighths, and twelfths, then label the sections. Students find equivalent fractions.	<i>First Steps in Mathematics: Number, Volume 1, pg 130.</i>
	4.2.F Fractions of a Collection (KU 3)	Students find different fractions of a collection and say which result in the same amount and which do not.	<i>First Steps in Mathematics: Number, Volume 1, pg 130.</i>
	4.2.E Comparing Lengths (KU 5)	Give each student a number line marked in units from 0-10. Ask students to draw a snake two and a quarter units long. Repeat this activity with numbers of different lengths.	<i>First Steps in Mathematics: Number, Volume 1, pg 138.</i>
	4.2.E Less Than 100 (KU 5)	Students think of the largest number they know that is less than 100. Students use grid paper to make a number line between 99 and 100. Students add numbers larger and justify positions on the number line.	<i>First Steps in Mathematics: Number, Volume 1, pg 139.</i>
	4.2.F & 4.2.G Ordering and Comparing Fractions (KU 5)	Ask students to use a half, a third, a quarter and three quarters as reference points to determine the size of a fraction, or to order and compare fraction numbers.	<i>First Steps in Mathematics: Number, Volume 1, pg 139.</i>

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	4.2.F Fraction Cards (KU 5)	Have students order sets of fraction cards with: <ul style="list-style-type: none"> ▪ Like denominators ▪ Like numerators 	<i>First Steps in Mathematics: Number, Volume 1, pg 139.</i>
	4.2.E Number Lines (KU 5)	Ask students to use strips of equal-length card and fold or mark it into fractional parts. Groups tape their fraction strips together to make separate number lines for halves, thirds, quarters, and so on. Then they label using fractions and mixed numbers.	<i>First Steps in Mathematics: Number, Volume 1, pg 139.</i>
	4.2.A Linking Decimals and Fractions (KU 6)	During classroom interaction and when students are using calculators, make the connection between a half and a decimal notation of 0.5.	<i>First Steps in Mathematics: Number, Volume 1, pg 146.</i>
	4.2.A Measurements (KU 6)	When students begin to write decimal notation in measurement activities, incorporate simple fractional language into the discussions.	<i>First Steps in Mathematics: Number, Volume 1, pg 147.</i>
	4.2.A Today's Number (KU 6)	Create a set of fraction and decimal cards from which students select "Today's Number". Discuss where students might see the number written in each way.	<i>First Steps in Mathematics: Number, Volume 1, pg 147.</i>
	4.2.A Concentration (KU 6)	Students make a matching set of cards (common fractions and equivalent decimal) to play "Concentration".	<i>First Steps in Mathematics: Number, Volume 1, pg 148.</i>
	4.2.A & 4.2.D Fractions to Decimals (KU 6)	Students carry out activities to explore how decimals are formed from a fraction.	<i>First Steps in Mathematics: Number, Volume 1, pg 150.</i>
	4.2.D Decimal Fractions (KU 6)	Students use square decimeters cut from millimeter grid paper as wholes, in order to show how unit fractions can be concretely converted to decimal fractions.	<i>First Steps in Mathematics: Number, Volume 1, pg 151.</i>
	4.2.D Matching Games (KU 6)	Students make sets of playing cards made up of pairs or sets of matching common fractions and decimals.	<i>First Steps in Mathematics: Number, Volume 1, pg 151.</i>

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	4.2.B & 4.2.E Ordering Collections (KU 6)	Students order collections of cards that show a mixture of common fractions and decimals. They then record them on in relative positions on a number line.	<i>First Steps in Mathematics: Number, Volume 1, pg 151.</i>
	4.2.A Measurements (KU 6)	Students use a standard tape measure to record tenths and hundredths on meter-long tapes.	<i>First Steps in Mathematics: Number, Volume 1, pg 151.</i>
	4.2.A, 4.2.B, & 4.2.D Fractional Measures (KU 6)	Each student cuts a strip of paper exactly one meter long and then folds their strips into halves, quarters, and so on.	<i>First Steps in Mathematics: Number, Volume 1, pg 151.</i>
	4.2.A 8.1 <i>Working with Tenths and the Area Model</i>	Writing the fraction name to match the shaded part of an area model.	<i>ORIGO Grade 4</i>
	4.2.A & 4.2.D 8.2 <i>Working with Tenths and the Linear Model</i>	Writing the fraction and decimal names using number lines.	<i>ORIGO Grade 4</i>
	4.2.A 8.3 <i>Developing a Picture of Hundredths</i>	Area model is used to break one whole into ten equal parts, and then each of the tenths into ten equal parts. The students describe the relationship between whole units, and tenths, and are introduced to the language of hundredths.	<i>ORIGO Grade 4</i>
	4.2.A 8.4 <i>Recording Hundredths</i>	Shading on a hundreds square to show hundredths and recording decimals involving hundredths.	<i>ORIGO Grade 4</i>
	4.2.A 8.5 <i>Writing Hundredths</i>	Students use a variety of methods to represent hundredths.	<i>ORIGO Grade 4</i>
	4.2.A <i>Session 10.1 Reading and Writing Hundredths</i>	Students extend their skills with decimals involving hundredths to examples that involve teens. They also analyze decimals with zeros in the tenths or hundredths place.	<i>ORIGO Grade 4</i>

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	4.2.A, 4.2.B, 4.2.H, 4.2.I <i>Session 10.2 Representing Decimals on a Number Line</i>	Students use a number line to find the nearest whole number to a decimal fraction and then figure out the distance to that whole number.	ORIGO Grade 4
	4.2.A <i>Session 10.3 Relating Common and Decimal Fractions</i>	Shading squares to represent fractions and using number lines.	ORIGO Grade 4
	4.2.D <i>Session 10.4 Matching Common Decimals and Fractions</i>	Students shade decimal squares and write the equivalent fraction.	ORIGO Grade 4
	4.2.D <i>Session 10.5 Using number lines to relate common and decimal fractions</i>	Writing equivalent decimals and fractions on a number line.	ORIGO Grade 4
	4.2.C <i>Session 8.2 Representing Fractions: Linear Model</i>	Students figure out methods to divide lengths into equal parts to show fractions.	ORIGO, Grade 5
	4.2.C, 4.2.F <i>Session 8.3 Relating Improper Fractions and Mixed Numbers</i>	Students explore different forms to write fractions greater than 1. They also investigate the concept of equivalence.	ORIGO Grade 5
	4.2.A, 4.2.C & 4.2.F <i>Session 8.4 Exploring Equivalence using paper strips</i>	Students fold paper rectangles and color strips or portions to describe the same part of a whole in different ways.	ORIGO, Grade 5
	4.2.A <i>Session 8.5 Relating Fractions to Tenths and Hundredths</i>	Students find tenths and hundredths expressed as common fractions that are equivalent to other given fractions.	ORIGO, Grade 5
	4.2.C <i>11.3 Finding Equivalent Fractions: Area Model</i>	Students use area models to formalize rules for finding equivalent fractions	ORIGO, Grade 5
	3.3.B & 4.2.E <i>Investigation 2: Models for Fractions Sessions 4 & 5: Fraction Tracks</i>	Students identify positions of fractions between 0 and 1 on a number line and labeling the fraction. They compare fractions with different denominators and use equivalencies to place fractions on a set of number lines from 0 to 1. They compare fractions up to $\frac{3}{2}$.	<i>Investigations: Name that Portion Grade 5, pg 46-52.</i>

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	4.2.A & 4.2.F <i>Fractions with a point</i>	Students expand knowledge of relationships between equivalent forms of fractions	<i>Navigating through Number & Operations, pg 41.</i>
	4.2.C <i>Put in Order</i> <i>(one class period; can be repeated multiple times during year)</i>	Students experience comparing and ordering fractions. This lesson focuses on students becoming more comfortable with fractional notation, learning to order fractions, and learning to use benchmarks of one-half and one.	<i>Lessons for Introducing Fractions -5, pg 105-115.</i>
	4.2.E Chapter 14: <i>Nicholas's Game</i> <i>(one class period + part of next day; lesson can be repeated multiple times.</i>	Students compare fractions and explain their reasoning. Students identify fractions that are less than, equal to, and greater than a particular starting fraction.	<i>Lessons for Introducing Fractions 4-5, pg 116-121..</i>
	4.2.E & 4.2. I <i>In-Between Fractions</i> <i>(two class periods + additional time for repeat experiences)</i>	Students use three fractions – $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$. “Is $\frac{1}{3}$ between $\frac{1}{2}$ and $\frac{1}{4}$?” Then “Name a fraction that is exactly halfway between $\frac{1}{2}$ and $\frac{1}{4}$.”	<i>Lessons for Extending Fractions, Grade 5, pg 62-74.</i>
	4.2.F & 4.2.G <i>Canceling Zeros</i> <i>(two class periods)</i>	This lesson give students valuable experience in thinking about equivalent fractions	<i>Lessons for Extending Fractions, Grade 5, pg 82-91</i>
	4.2.F <i>Fraction CDs</i>	To construct and use a model that illustrates the meaning of common denominators. To use the model to understand and practice addition of like and unlike fractions.	<i>Fabulous Fractions, AIMS, pg.20-28.</i>
	3.3.A, 3.3.D, 4.2.F, 4.2.I <i>Chocolate Bars (KU 6)</i>	Repeat practice predicting and testing sharing a collection of pretend chocolate bars between various numbers of people until students are able to write their own rule for working out what fraction of bar each person will get.	<i>First Steps in Mathematics: Number, Volume 1, pg 147.</i>
	4.2.D & 4.2.F <i>Matching Games (KU 7)</i> <i>(leave out %)</i>	Invite students to make sets of playing cards made up of pairs or sets of matching common fractions and decimals.	<i>First Steps in Mathematics: Number, Volume 1, pg 158.</i>
	4.2.C <i>Comparisons and Equivalencies</i> <i>Session 6: Counting and Equivalency</i>	Involve students in counting fractions beyond one.	<i>Making Sense of Fractions pg 25</i>

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	<p>4.2.D <i>What's the Point?</i></p>	<p>To represent and interpret fractions in a base-ten circle model To relate fractions, decimals, and percents To find equivalent representations.</p>	<p><i>Fabulous Fractions, AIMS, pg. 29-33.</i></p>
	<p>4.2.E <i>Show Me the Money</i></p>	<p>To understand relative magnitude of decimals To practice ordering decimals To experience coins as a model for understanding decimals as base-ten fractions.</p>	<p><i>Fabulous Fractions, AIMS, pg. 34-36.</i></p>
	<p>4.2.I Talking in Circles</p>	<p>To recognize fractions in real-world problems or situations. To represent and interpret fractions in a circle or pie graph To recognize the relative size of fractions</p>	<p><i>Fabulous Fractions, AIMS, pg. 78-81.</i></p>
	<p>Poster Proofs</p>	<p>To recognize fractions in a variety of equivalent forms To be able to show the relationship of two unequal fractions using a variety of models and examples.</p>	<p><i>Fabulous Fractions, AIMS, pg. 101-102.</i></p>
	<p>4.2.I Sharing Paper (KU 6)</p>	<p>Students work in groups to share strips of paper equally.</p>	<p><i>First Steps in Mathematics: Number, Volume 1, pg 147</i></p>
	<p>3.3.D & 4.2.I Chocolate Bars (KU 6)</p>	<p>Students use pretend chocolate bars made from rectangles of paper to model sharing two bars among three people. Activity is repeated for different numbers of bars and different numbers of people. The link between division and fractions is made.</p>	<p><i>First Steps in Mathematics: Number, Volume 1, pg 149.</i></p>
	<p>4.2.I Pizza Problems (KU 6)</p>	<p>Problems are posed to students to help them relate division to fractional notation.</p>	<p><i>First Steps in Mathematics: Number, Volume 1, pg 149.</i></p>
	<p>4.2.I Fair Shares (KU 6)</p>	<p>Extension of the Pizza Problem above. This activity draws out the idea that if anything is shared between three people they each get one-third.</p>	<p><i>First Steps in Mathematics: Number, Volume 1, pg 149.</i></p>

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	4.2.I Fifths (KU 6)	Students draw pictures to show that two fifths of one is the same as one fifth of two.	<i>First Steps in Mathematics: Number, Volume 1, pg 149.</i>
<p>Computation of Fractions *Dangerous Rush to Rules Use Problem-Based, Number Sense Approach Addition and Subtraction</p> <ul style="list-style-type: none"> ▪ Informal Exploration <p>Watch out for the myth of common denominators</p>			<i>Van De Walle pg 229 (265)</i>
<p>Operating Phase During the Operating Phase, students see how the intervals between whole numbers can be split and re-split into increasingly smaller intervals and realize the significance of the relationship between successive places.</p>			<i>First Steps: Diagnostic Map - Number</i>
<p>5.2.A Represent addition and subtraction of fractions and mixed numbers using visual and numerical models and connect the representation to the related equation. 5.2.B Represent addition and subtraction of decimals using place value models and connect the representation to the related equation</p>			<i>Mathematics Standards</i>
	5.2.A <i>Investigation 1: Session 5 Combining Fractions in a Design</i>	Students explore relationships among halves, fourths, and eighths and combine them to make a whole. Students write equations that reflect pictures with halves, fourths, and eighths.	<i>Investigations: Different Shapes, Equal Pieces 4th, pg 19</i>
	5.2.A <i>Investigation 2: Session 3</i>	Combining 1/3s, 1/6s, 1/12s using dot paper to find combinations of fractions to equal one whole.	<i>Investigations: Different Shapes, Equal Pieces 4th</i>
	4.2.A & 5.2A <i>Investigation 2: Models for Fractions Session 1 & 2: Fractions on Clocks</i>	Students represent fractions as rotation around a circle. Students add fractions.	<i>Investigations: Name that Portion Grade 5, pg 34-40</i>
	2.4.F, 3.3.C, 4.2.E, & 5.2.A <i>Investigation 2: Models for Fractions Session 3: Fraction Strips</i>	Students partition paper strips into halves, thirds, fourths, and sixths. Students use equivalents to partition fraction strips. They compare fractions of different denominators. Using the fraction strips to show equivalent fractions and addition of fractions	<i>Investigations: Name that Portion Grade 5, pg 41-45.</i>

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	4.2.F, 4.2.E, 5.2.A <i>Investigation 2: Models for Fractions</i> <i>Session 6: The Fraction Track Game</i>	Students find equivalent fractions, order fractions, add fractions and break fractions into parts while playing a game.	<i>Investigations: Name that Portion</i> <i>Grade 5, pg 53-57.</i>
	4.2.F, 4.2.E, 5.2.A <i>Investigation 2: Models for Fractions</i> <i>Session 7 & 8: Fraction Games</i>	Students find equivalent fractions, order fractions, add fractions and break fractions into parts while playing a game.	<i>Investigations: Name that Portion</i> <i>Grade 5, pg 58-61..</i>
	5.2.B <i>Session 2.1 Working with Decimals – Using Place Value to Record Numbers Involving Thousands</i>	The area model is used to introduce the place-value aspect of thousandths and its relationship to tenths and hundredths.	<i>ORIGO</i> <i>Grade 5</i>
	5.2.A Ways to Share (KU 6)	Extend sharing activities by asking students to share three pizzas between four people in a number of different ways.	<i>First Steps in Mathematics: Number, Volume 1, pg 150.</i>
	5.2.A More Sharing (KU 6)	Brainstorm with students situations where food is sliced into set portions.	<i>First Steps in Mathematics: Number, Volume 1, pg 150.</i>
	5.2.A Sensible Fractions (KU 7)	Pose some problems and have students say when adding fractions makes sense.	<i>First Steps in Mathematics: Number, Volume 1, pg 157.</i>
	5.2.B <i>Session 2.2 Relating Tenths, Hundredths, and Thousandths</i>	Students use an area model and numeral expanders to reinforce the relationship between tenths, hundredths, and thousandths.	<i>ORIGO</i> <i>Grade 5</i>
	5.2.B <i>Session 2.3 Working with Place Value</i>	Students use an abacus to study the places from thousands to thousandths.	<i>ORIGO</i> <i>Grade 5</i>
	5.2.B <i>Session 2.4 Comparing Tenths and Hundredths</i>	A number line is used to compare decimal fractions to two decimal places. The number line model naturally encourages students to compare numbers starting with places with the greatest value.	<i>ORIGO</i> <i>Grade 5</i>
	4.2.H & 5.2.B <i>Session 2.5 Using Relative Position to Round Decimals</i>	Students identify the whole number or tenth that is closest to a decimal involving tenths or hundredths. Number line model is used.	<i>ORIGO</i> <i>Grade 5</i>

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	5.2.A <i>Session 11.1 Adding Common Fractions: Same Denominator</i>	Students use area model to add proper fractions.	<i>ORIGO Grade 5</i>
	5.2.A <i>Session 11.2 Subtracting Common Fractions: Same Denominator.</i>	Students use area model to subtract proper fractions	<i>ORIGO Grade 5</i>
	5.2.A <i>Session 11.3 Finding Equivalent Fractions: Area Model</i>	Students use a number line and fraction strips to find equivalent fractions to add pairs of fractions.	<i>ORIGO Grade 5</i>
	5.2.A & 5.2.C <i>Session 11.4 Adding Common Fractions: Related Denominator</i>	Students use a number line and fraction strips to find equivalent fractions to subtract pairs of fractions.	<i>ORIGO Grade 5</i>
	5.2.A & 5.2.C <i>Session 11.5 Subtracting Common Fractions: Related Denominators</i>	Students use number lines and then fraction strips to help find equivalent fractions so they can subtract one fraction from another.	<i>ORIGO Grade 5</i>
	5.2.B <i>Session 2.1 Working with Decimals – Using Place Value to Record Numbers Involving Thousands</i>	The area model is used to introduce the place value aspect of thousandths and its relationship to tenths and hundredths.	<i>ORIGO Grade 5</i>
	5.2.A <i>Slide Ruler Fractions</i> Parallel number lines with unit fractions and their multiples Understanding relative size of fractions on a number line.	To order fractions on a number line To experience addition and subtraction of fractions on a number line To connect fractions and linear measurement.	<i>Fabulous Fractions, AIMS, pg. 46-52.</i>
	4.2.F, 4.2.I & 5.2.A <i>Only One – Fraction Kit Activities (three class periods)</i>	This lesson describes four activities that promote student's understanding of equivalence and combining fractions.	<i>Lessons for Introducing Fractions 4-5, pg 122-132.</i>

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	<i>Introducing Addition and Subtraction (one class period to introduce + time for additional experiences over several days)</i>	Introduces the addition and subtraction of fractions and mixed numbers. Lesson builds on student's understanding of equivalent fractions and mixed numbers and finding differences rather than teaching algorithms.	<i>Lessons for Extending Fractions, Grade 5, pg 92-101.</i>
	5.2.A <i>Problems with Oranges (one class period)</i>	Students consider several variations of the same problem in which friends share oranges. Students present their strategies for figuring out how many oranges.	<i>Lessons for Extending Fractions, Grade 5, pg 102-116.</i>
	5.2.A <i>Make a One (at least two class periods)</i>	A problem solving experience that provides practice with combining fractions.	<i>Lessons for Extending Fractions, Grade 5, pg 117-127.</i>
	5.2.A <i>Fraction Capture (two class periods + additional time for playing)</i>	A two-person game that gives students practice with representing and combining fractions. Students sketch and shade a pie-shaped piece to represent that fractional part.	<i>Lessons for Extending Fractions, Grade 5, pg 128-143.</i>

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Developing the Algorithm			<i>Van De Walle</i>
<ul style="list-style-type: none"> ▪ Like denominators ▪ Unlike denominators ▪ Common multiples ▪ Mixed Numbers 			
<p>5.2.F Fluently and accurately add and subtract decimals.</p> <p>5.2.G Estimate sums and differences of fractions, mixed numbers, and decimals to approximate solutions to problems and determine reasonableness of answers.</p> <p>5.2.H Solve single- and multi-step word problems involving addition and subtraction of whole numbers, fractions (including mixed numbers), and decimals, and verify the solutions.</p>			<i>Mathematics Standards</i>
	13.1 Running Through the Multiples		<i>Van De Walle pg 232</i>
	13.2 LCM Flash Cards		<i>Van De Walle pg 232</i>
	<p>5.2.H <i>Investigation 2: Models for Fractions</i> <i>Session 9: Problems with Fractions</i></p>	<p>Students solve word problems using fractions and percents and later share their solution strategies. They find fractional parts of whole numbers, compute with fractions, make sense of word problems, and compare fractions.</p>	<i>Investigations: Name that Portion Grade 5, pg 62-63.</i>
	<p>5.2.F <i>Session 1.4 Adding Dollars and Cents: Rounding and Adjusting</i></p>	<p>Students add prices that are close to whole dollar amounts. They use the whole dollars to find the total and then make adjustments to figure out the exact amount.</p>	<i>ORIGO Grade 4</i>
	<p>5.2.F <i>Session 3.5 Subtracting Dollars and Cents</i></p>	<p>Students use count-on strategy to figure out the amount of money they have left after a purchase.</p>	<i>ORIGO Grade 4</i>
	<p>5.2.F <i>Session 3.2 Adding Dollars and Cents</i></p>	<p>The split strategy is applied to examples involving dollars and cents that do not require bridging over a whole dollar. All the values are multiples of five cents.</p>	<i>ORIGO Grade 5</i>

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	5.2.F & 5.2 G <i>Session 3.3 Rounding and Adjusting to Add Ones and Tenths</i>	Students round one or both decimals involving tenths before adding the numbers and adjusting the answer.	<i>ORIGO Grade 5</i>
	5.2.F & 5.2 G <i>Session 3.4 Rounding and Adjusting to Add Dollars and Cents</i>	Students use a round-and-adjust strategy to add prices that are close to a whole number.	<i>ORIGO Grade 5</i>
	5.2.F <i>Session 3.5 Using Compatible Numbers to Add Ones and Tenths</i>	Students utilize compatible decimal fractions to add mentally. Two decimals are considered to be compatible if their total is a whole number.	<i>ORIGO Grade 5</i>
	5.2.F <i>Session 5.4 Subtracting Numbers Involving Tenths</i>	Students are encouraged to split a number into a whole number and tenths and then subtract each part using thinking that matches jumps along a number line.	<i>ORIGO Grade 5</i>
	5.2.F <i>Using Formal Methods to Add and Subtract Decimals</i> <i>Session 9.1 Adding Dollars and Cents</i>	Students extend written addition to four-digit amounts involving dollars and cents. They are encouraged to add mentally whenever possible.	<i>ORIGO Grade 5</i>
	5.2.F <i>Session 9.2 Adding More Than Two Decimals: Tenths and Hundredths</i>	Students extend decimal addition to more than three addends of two decimal places.	<i>ORIGO Grade 5</i>
	5.2.F <i>Session 9.3 Subtracting Tenths</i>	Students investigate subtraction of decimals involving tenths.	<i>ORIGO Grade 5</i>
	5.2.F <i>Session 9.3 Subtracting Dollars and Cents</i>	Students extend written subtraction to amounts involving dollars and cents.	<i>ORIGO Grade 5</i>
	5.2.F <i>Session 9.5 Subtracting Tenths and Hundredths</i>	Students develop written methods to solve subtraction problems. The examples involve finding a missing addend and the difference.	<i>ORIGO Grade 5</i>
	5.2.E Who Has? For Fractions	A game for whole class. Students work on fluently adding and subtracting fractions and determining equivalent fractions.	<i>Fabulous Fractions, AIMS, pg. 95-99.</i>

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	5.2.E Chapter 13: <i>Score the Difference</i>	This game gives students practice with adding and subtracting fractions.	<i>Lessons for Extending Fractions 5, pg 144-151.</i>
	5.2.E Chapter 14: <i>The Smaller Answer Wins (one class period to introduce the game + additional time to play)</i>	Two person game that combines luck and strategic thinking to give students practice comparing and adding fractions.	<i>Lessons for Extending Fractions, Grade 5, pg 152-162.</i>
	12.14 Pick the Best Estimate sums and differences	Refine the estimates to using harder sums and differences	<i>Van De Walle pg 221 and 232</i>
	5.6 Tangrammy Squares	To use tangram pieces as an area model for study of fractions To explore the fractional relationships among the tangram pieces.	<i>Fabulous Fractions, AIMS, pg. 75-77.</i>
	5.2.H <i>Adding and Subtracting Fractions: Session 1: Problem Solving</i>	Create addition/subtraction problems for students to model, solve and record.	<i>Making Sense of Fractions 3-6, p g 30.</i>
	5.2.G & 5.2.H <i>Adding and Subtracting Fractions: Session 2: In Order</i>	Students will estimate and sequence sums and differences.	<i>Making Sense of Fractions 3-6, p g 31.</i>
	5.2.E <i>Adding and Subtracting Fractions: Session 3: Fraction Game: What You Spin is What You Take/Take Away</i>	Students practice fluently adding and subtracting fractions, including mixed numbers. Students can also play the game <i>Thirds, Sixths and Twelfths</i> starting with different target numbers.	<i>Making Sense of Fractions 3-6, p g 32-33..</i>
	5.2.F & 5.2.G <i>Adding and Subtracting Fractions: Session 4: Menu Options</i>	Menu tasks are designed for partner work. Students work on estimating and adding and subtracting fractions.	<i>Making Sense of Fractions 3-6, p g 34.</i>
	5.2.H <i>Adding and Subtracting Fractions: Session Fraction Problems and Riddles</i>	Students work in pairs to solve fraction problems, record their solutions and verify the solutions by sharing.	<i>Making Sense of Fractions 3-6, p g 37.</i>

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<p>Estimation and Simple Methods</p> <p>Multiplication</p> <p>Informal exploration</p> <ul style="list-style-type: none">▪ Beginning Concepts▪ Unit Parts without Subdivisions <p>Subdividing the unit parts</p> <p>Developing the Algorithm</p> <ul style="list-style-type: none">▪ Beginning▪ Factors > 1 <p>Mental techniques</p> <p>Division</p> <p><i>Informal Exploration: Partition Concept</i></p> <ul style="list-style-type: none">▪ Whole number divisors▪ Fractional Divisors <p><i>Informal Exploration: Measurement Concept</i></p> <ul style="list-style-type: none">▪ Whole number results <p>Answers that are not whole numbers</p> <p>Developing the Algorithms</p> <ul style="list-style-type: none">▪ Common denominator▪ Invert and multiply <p>Curricular Decisions</p>	<p><i>Van De Walle</i></p>
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Research has identified several different interpretations (or constructs) of rational numbers often summarized as:

1. The *part-whole* interpretation depends on the ability to partition either a continuous quantity (including area, length, and volume models) or a set of discrete objects into equal sized subparts or sets.
2. A fraction can represent a *measure* of a quantity relative to one unit of that quantity.
3. A fraction a/b may represent the operation of *division*. The division or quotient interpretation may be understood through partitioning and equal sharing.
4. A fraction can be used as an *operator* to operate on a unit (or on the result of a previous operation) such as $\frac{3}{4}$ of $12 = 9$. The misconception that multiplication “always makes bigger” and division “always makes smaller” is common.
5. Fractions can be used as a *ratio* – a method of comparing the sizes of two sets or two measurements.

Ten tips to make fractions more readily understandable for students:

1. Give a greater emphasis to the meaning of fractions than on procedures for manipulating them.
2. Develop a generalizable rule for explaining the numerator and denominator of a fraction.
3. Emphasize that fractions are numbers, making extensive use of number lines in representing fractions and decimals.
4. Take opportunities early to focus on improper fractions and equivalences.
5. Provide a variety of models to represent fractions.
6. Link fractions to key benchmarks, and encourage estimation
7. Give emphasis to fractions as division.
8. Link fractions, decimals, and percents whenever possible

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9. Take the opportunity to interview several students one on one on the kinds of tasks discussed in this article to gain awareness of their thinking and strategies.
10. Look for examples and activities that can engage students in thinking about fractions in particular and rational numbers in general.